ESERCIZI PROPOSTI

1. *Ridurre allo stesso indice i seguenti radicali.*

- $\sqrt{3}, \sqrt[4]{5}, \sqrt[6]{2}$ $\left[\mathbf{R}.\ \sqrt[12]{3^6}, \sqrt[12]{5^3}, \sqrt[12]{2^2}\right]$
- $\sqrt{2}, \sqrt[3]{5}, \sqrt[4]{10}, \sqrt[6]{6}$ $\left[\mathbf{R}.\ \sqrt[12]{2^6}, \sqrt[12]{5^4}, \sqrt[12]{10^3}, \sqrt[12]{6^2}\right]$
- $\sqrt[5]{b^3}, \sqrt[6]{b^5}, \sqrt[10]{b^6},\ \ b \geq 0$ $\left[\mathbf{R}.\ \sqrt[30]{b^{18}}, \sqrt[30]{b^{25}}, \sqrt[30]{b^{18}}\right]$

2. *Scrivere le seguenti espressioni sotto forma di un unico radicale, dopo aver determinato le condizioni di esistenza.*

- $3\sqrt[4]{5}$ $\left[\mathbf{R}.\ \sqrt[4]{405}\right]$
- $a^3 \cdot \sqrt{a}$ $\left[\mathbf{R}.\ \sqrt[3]{a^7}\right]$
- $\dfrac{3b^2 x}{y^2}\sqrt{\dfrac{xy}{b^3}}$ $\left[\mathbf{R}.\ \sqrt{\dfrac{9bx^3}{y^3}}\right]$
- $(x-y)\sqrt[4]{x^2-y^2}$ $\left[\mathbf{R}.\ \sqrt[4]{(x-y)^5\cdot(x+y)}\right]$

3. *Portare fuori segno di radice i fattori il cui esponente è maggiore o uguale all'indice della radice, dopo aver determinato le condizioni di esistenza:*

- $\sqrt{243}$ $\left[\mathbf{R}.\ 9\sqrt{3}\right]$
- $\sqrt[3]{x^4y^2z^5}$ $\left[\mathbf{R}.\ x\cdot z\sqrt[3]{xy^2z^2}\right]$
- $\sqrt{a^2x^2+a^2x^6}$ $\left[\mathbf{R}.\ |a|\cdot|x|\sqrt{1+x^4}\right]$
- $\sqrt{x^5-2x^4+x^3}$ $\left[\mathbf{R}.\ |x|\cdot|x-1|\sqrt{x}\right]$
- $\sqrt{\dfrac{x^4-2x^2y^2+y^4}{x-y}}$ $\left[\mathbf{R}.\ |x+y|\sqrt{x-y}\right]$
- $\sqrt[3]{\dfrac{x^4-2x^3+2x-1}{x^3+1}}$ $\left[\mathbf{R}.\ (x-1)\sqrt[3]{\dfrac{1}{x^2-x+1}}\right]$

4. *Determinare il dominio delle seguenti funzioni.*

- $f(x)=\sqrt{9-x^2}+\sqrt[3]{\dfrac{1}{x}}$ $\left[\mathbf{R}.\ [-3,0)\cup(0,3]\right]$
- $f(x)=\sqrt[4]{x-3}\cdot\sqrt[4]{x+5}$ $\left[\mathbf{R}.\ [3,+\infty)\right]$
- $f(x)=\sqrt[4]{x^2+2x-15}$ $\left[\mathbf{R}.\ (-\infty,-5]\cup(3,+\infty]\right]$
- $f(x)=\sqrt{\dfrac{1}{x-4}}+\sqrt[6]{x^2-1}-\sqrt[3]{\dfrac{1}{x^2+1}}$ $\left[\mathbf{R}.\ (4,+\infty)\right]$

5. Risolvere algebricamente e, dove è possibile, graficamente le seguenti equazioni e disequazioni irrazionali.

- $\sqrt{x+3} = 1 - 3x$ $\left[\mathbf{R}.\ x = -\frac{2}{9}\right]$
- $\sqrt[3]{x+4} = 2$ $\left[\mathbf{R}.\ x = 4\right]$
- $\sqrt[4]{x} = -3$ $\left[\mathbf{R}.\ \varnothing\right]$
- $\sqrt{(x-2)(x+3)} + \sqrt[4]{x^2 - 4} = 0$ $\left[\mathbf{R}.\ x = 2\right]$
- $\sqrt{x-1} = \sqrt{2x-3}$ $\left[\mathbf{R}.\ x = 2\right]$
- $\sqrt{9 - x^2} = 2\sqrt{2}$ $\left[\mathbf{R}.\ x = \pm 1\right]$
- $\sqrt{x^2 + 1} = 1 - x$ $\left[\mathbf{R}.\ x = 0\right]$
- $\sqrt{2-x} \leq 1$ $\left[\mathbf{R}.\ [1,2]\right]$
- $\sqrt{x-4} > 0$ $\left[\mathbf{R}.\ (4,+\infty)\right]$
- $\sqrt{x+1} < 2x - 1$ $\left[\mathbf{R}.\ \left(\frac{5}{4},+\infty\right)\right]$

- $\sqrt[3]{x-2}+\sqrt[3]{x-1}<0$ $\left[\mathbf{R}.\left(-\infty,\frac{3}{2}\right)\right]$
- $\sqrt{\frac{x-4}{x+2}}<2$ $\left[\mathbf{R}.\left(-\infty,-4\right)\cup\left[4,+\infty\right)\right]$
- $\sqrt{2-x}\geq 2x-3$ $\left[\mathbf{R}.\left(-\infty,\frac{7}{4}\right]\right]$
- $\sqrt{4x-1}\geq\sqrt{x-6}$ $\left[\mathbf{R}.\left[6,+\infty\right)\right]$
- $\sqrt{-\left(x^2-5x+4\right)}\geq x-1$ $\left[\mathbf{R}.\left[1,\frac{5}{2}\right]\right]$
- $\sqrt[3]{1+\frac{1}{2}x}\leq\frac{1}{2}x+1$ $\left[\mathbf{R}.\left[-4,-2\right]\cup\left[0,+\infty\right)\right]$
- $\frac{\sqrt{x}-1}{\sqrt{|x|}-2}\geq 0$ $\left[\mathbf{R}.\left[0,1\right]\cup\left(4,+\infty\right)\right]$
- $\sqrt{|x+2|}\geq 2$ $\left[\mathbf{R}.\left(-\infty,-6\right]\cup\left[2,+\infty\right)\right]$