ESERCIZI PROPOSTI

1. *Applicando le proprietà dei logaritmi trasformare le seguenti espressioni in somme algebriche*:

- $\ln\left(\dfrac{2\cdot x\cdot y}{\sqrt{z}}\right)$ con $x,y,z>0$ $\left[\mathbf{R}.\ \ln 2+\ln x+\ln y-\dfrac{1}{2}\ln z\right]$
- $\ln\left(\sqrt[3]{\dfrac{x^2-y^2}{x}}\right)$ con $-x<y<x$ $\left[\mathbf{R}.\ \dfrac{1}{3}\ln(x-y)+\dfrac{1}{3}\ln(x+y)-\dfrac{1}{3}\ln x\right]$
- $\ln\sqrt{(x-y)\sqrt{x+y}}$ con $-x<y<x$ $\left[\mathbf{R}.\ \dfrac{1}{2}\ln(x-y)+\dfrac{1}{4}\ln(x+y)\right]$

2. *Applicando le proprietà dei logaritmi scrivere ciascuna delle seguenti espressioni sotto forma di un unico logaritmo.*

- $2\ln x-4\ln y+\dfrac{1}{2}\ln z$ con $x,y,z>0$ $\left[\mathbf{R}.\ \ln\dfrac{x^2\sqrt{z}}{y^4}\right]$
- $2\ln(x-3)-\ln\left(x^2-9\right)+\dfrac{1}{2}\ln(x+1)$ con $x>3$ $\left[\mathbf{R}.\ \ln\dfrac{(x-3)\sqrt{x+1}}{x+3}\right]$
- $\dfrac{1}{4}\ln x-3\ln 5+7\ln y-2\ln z$ con $x,y,z>0$ $\left[\mathbf{R}.\ \ln\dfrac{\sqrt[4]{x}\cdot y^7}{125z^2}\right]$

3. Stabilire per quali valori di x sono vere le seguenti uguaglianze:

- $\ln\left(2x^2-5x+2\right)=\ln(x-2)+\ln(2x-1)$ $\left[\mathbf{R}.\ (2,+\infty)\right]$
- $\ln(x-1)^2=2\ln(x-1)$ $\left[\mathbf{R}.\ (1,+\infty)\right]$
- $\ln(x-1)^2=2\ln|x-1|$ $\left[\mathbf{R}.\ (-\infty,1)\cup(1,+\infty)\right]$
- $\ln\dfrac{2x-5}{1-x}=\ln 2+\ln\left(x-\dfrac{5}{2}\right)-\ln(1-x)$ $\left[\mathbf{R}.\ \varnothing\right]$
- $\ln\dfrac{2x-5}{1-x}=\ln 2+\ln\left(\dfrac{5}{2}-x\right)-\ln(x-1)$ $\left[\mathbf{R}.\ \left(1,\dfrac{5}{2}\right)\right]$

4. Risolvere le seguenti equazioni e disequazioni esponenziali e logaritmiche.

- $5^x=3^{x-1}$ $\left[\mathbf{R}.\ x=\dfrac{\ln 3}{\ln 3-\ln 5}\right]$
- $4\cdot 5^{x+1}=3\cdot 2^{2x+3}$ $\left[\mathbf{R}.\ x=\dfrac{\ln 6/5}{\ln 5/4}\right]$
- $\left|2^{x+1}-1\right|=7$ $\left[\mathbf{R}.\ x=2\right]$

- $3^{3x-x^2} \leq 9$ $\left[\mathbf{R}.\ (-\infty,1] \cup [2,+\infty)\right]$
- $\left(\frac{1}{2}\right)^{x+4} > 1$ $\left[\mathbf{R}.\ (-\infty,-4)\right]$
- $9^x - 12 \cdot 3^x + 27 \leq 0$ $\left[\mathbf{R}.\ [1,2]\right]$
- $7^{2x} - 5 \cdot 7^x + 6 > 0$ $\left[\mathbf{R}.\ (-\infty,\log_7 2) \cup (\log_7 3,+\infty)\right]$
- $\dfrac{2e^{-x} + e^x - 3}{1 - e^x} \geq 0$ $\left[\mathbf{R}.\ (-\infty,0) \cup (0,\ln 2]\right]$
- $\log_x \dfrac{5}{3} = -2$ $\left[\mathbf{R}.\ x = \sqrt{\dfrac{3}{5}}\right]$
- $\log_{10}(2x+3) + 1 = 2\log_{10}(2x+5)$ $\left[\mathbf{R}.\ x = \pm\dfrac{\sqrt{5}}{2}\right]$
- $\ln^3 x - \ln^2 x = 0$ $\left[\mathbf{R}.\ x = 1 \cup x = e\right]$
- $\ln(9-x) < 0$ $\left[\mathbf{R}.\ (8,9)\right]$
- $\log_{\frac{1}{2}}(3x-5) - 2 > 0$ $\left[\mathbf{R}.\ \left(\dfrac{5}{3},\dfrac{7}{4}\right)\right]$

- $\log_{\frac{1}{5}} \dfrac{x+1}{x-1} < \log_{\frac{1}{5}} x$ $\left[\mathbf{R}.\left(1, 1+\sqrt{2}\right)\right]$

- $\log_4{}^2(x+2) - 3\log_4(x+2) - 4 > 0$ $\left[\mathbf{R}.\left(-2, -\dfrac{7}{4}\right) \cup (254, +\infty)\right]$

- $|1 - \log_3 x| < 1 + \log_3 x$ $\left[\mathbf{R}.\left(1, +\infty\right)\right]$

- $\log_{\frac{1}{2}}\left(x^2 - 9\right) > 0$ $\left[\mathbf{R}.\left(-\sqrt{10}, -3\right) \cup \left(3, \sqrt{10}\right)\right]$

- $\dfrac{3 - \log_{\frac{1}{2}} x}{4 + \log_2 x} \leq 0$ $\left[\mathbf{R}.\left(\dfrac{1}{16}, \dfrac{1}{8}\right]\right]$