



ESERCIZI PROPOSTI

1. Calcolare i valori delle seguenti espressioni:

- $5\sin\frac{p}{2} + \frac{2}{3}\operatorname{tg}\frac{p}{4} - \frac{1}{4}\cos\frac{p}{3} + 5\cos(p)$ $\left[\text{R. } \frac{13}{24} \right]$
- $\sqrt{3}\sin\frac{p}{3} - \cos\frac{5p}{6} + \frac{1}{\sqrt{2}}\cos\left(-\frac{p}{4}\right) + 2\operatorname{tg}\frac{p}{3}$ $\left[\text{R. } \frac{4+5\sqrt{3}}{2} \right]$
- $\cos\frac{2p}{3} - \sin\left(-\frac{5p}{6}\right) - \operatorname{tg}\frac{p}{4} + \operatorname{tg} - (3p) + \operatorname{tg}\left(-\frac{7p}{6}\right)$ $\left[\text{R. } -\frac{3+\sqrt{3}}{3} \right]$

2. Verificare le seguenti identità

- $\cos\left(\frac{p}{2} - x\right) + \cos(p - x) + \sin(p + x) = -3\cos x + 2\cos(-x)$
- $\sin\left(\frac{p}{2} + x\right)\cos(p - x) = \sin\left(\frac{p}{2} + x\right)\cos(p + x)$
- $\cos(2x) = 1 - 2\sin^2 x$
- $\cos(2x) = 2\cos^2 x - 1$
- $\operatorname{tg}\left(\frac{p}{4} - x\right) \cdot \operatorname{tg}\left(\frac{p}{4} + x\right) = 1$



9 Funzioni Trigonometriche

- $\operatorname{tg}^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$
- $\cos^2 x = \frac{1}{1 + \operatorname{tg}^2 x}$
- $\sin^2 x = \frac{\operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}$
- $\frac{\cos(2x)}{1 + \sin(2x)} = \frac{\cotg x - 1}{\cotg x + 1}$
- $\cos\left(\frac{p}{3} - x\right) + \cos\left(\frac{p}{3} + x\right) = \cos x$

3. Risolvere in \mathbf{R} le seguenti equazioni:

- $2 \cos x = -1$
 $\left[\mathbf{R}. x = \pm \frac{2}{3}p + 2kp \right]$
- $2 \sin(2x) = \sqrt{3}$
 $\left[\mathbf{R}. x = \frac{p}{3} + kp \cup x = \frac{p}{6} + kp \right]$
- $2 \cos^2 x = 1$
 $\left[\mathbf{R}. x = \pm \frac{p}{4} + 2kp \cup x = \pm \frac{3p}{4} + 2kp \right]$



9 Funzioni Trigonometriche

- $\sin\left(\frac{p}{4} - x\right) = \frac{\sqrt{3}}{2}$
 $\left[\mathbf{R.} \ x = -\frac{p}{12} + 2k\mathbf{p} \cup x = -\frac{5p}{12} + 2k\mathbf{p} \right]$
- $\cos\left(x + \frac{p}{6}\right) = \cos\left(2x - \frac{p}{4}\right)$
 $\left[\mathbf{R.} \ x = \frac{5p}{12} + 2k\mathbf{p} \cup x = -\frac{p}{36} + k\frac{2p}{3} \right]$
- $\operatorname{tg}\left(\frac{p}{6} - 2x\right) = \operatorname{tg}\left(3x - \frac{p}{3}\right)$
 $\left[\mathbf{R.} \ x = \frac{p}{10} + k\frac{p}{5} \right]$
- $6\sin^2 x - 13\sin x + 5 = 0$
 $\left[\mathbf{R.} \ x = \frac{p}{6} + 2k\mathbf{p} \cup x = \frac{5p}{6} + 2k\mathbf{p} \right]$
- $2\cos^2(2x) + \cos(2x) = 0$
 $\left[\mathbf{R.} \ x = \frac{p}{4} + k\frac{p}{2} \cup x = \pm\frac{p}{3} + k\mathbf{p} \right]$
- $\sin x + \cos x - 1 = 0$
 $\left[\mathbf{R.} \ x = \frac{p}{2} + 2k\mathbf{p} \cup x = 2k\mathbf{p} \right]$
- $\sqrt{3}\sin x = \cos x$
 $\left[\mathbf{R.} \ x = \frac{p}{6} + k\mathbf{p} \right]$
- $4\sin^2 x - 2\sqrt{3}\sin x \cos x - 2\cos^2 x - 1 = 0$
 $\left[\mathbf{R.} \ x = -\frac{p}{6} + k\mathbf{p} \cup x = \frac{p}{3} + k\mathbf{p} \right]$
- $2\sin^2 x - \cos x = 1$
 $\left[\mathbf{R.} \ x = \pm\frac{p}{3} + 2k\mathbf{p} \cup x = \mathbf{p} + 2k\mathbf{p} \right]$



9 Funzioni Trigonometriche

4. Risolvere le seguenti disequazioni negli intervalli a fianco indicati:

- | | | |
|--|--|---|
| • $2\sin x > \sqrt{3}$ | $[0, 2p]$ | $\left[\mathbf{R.} \left[0, \frac{p}{3} \right) \cup \left(\frac{2p}{3}, 2p \right] \right]$ |
| • $\cos x > p$ | $[0, 2p]$ | $\left[\mathbf{R.} \emptyset \right]$ |
| • $\cos x \geq -\frac{1}{2}$ | $[-p, p]$ | $\left[\mathbf{R.} \left[-\frac{2p}{3}, \frac{2p}{3} \right] \right]$ |
| • $\operatorname{tg} x \leq -1$ | $[0, p]$ | $\left[\mathbf{R.} \left(\frac{p}{2}, \frac{3p}{4} \right) \right]$ |
| • $3\operatorname{tg}^2 x < 1$ | $\left(-\frac{p}{2}, \frac{p}{2} \right)$ | $\left[\mathbf{R.} \left(-\frac{p}{6}, \frac{p}{6} \right) \right]$ |
| • $\cos^2 x \leq \frac{3}{4}$ | $[0, 2p]$ | $\left[\mathbf{R.} \left[\frac{p}{6}, \frac{5p}{6} \right] \cup \left[\frac{7p}{6}, \frac{11p}{6} \right] \right]$ |
| • $2\cos^2 x - 5\cos x - 3 > 0$ | $[0, 2p]$ | $\left[\mathbf{R.} \left[0, \frac{p}{3} \right) \cup \left(\frac{5p}{3}, 2p \right] \right]$ |
| • $2\sin x \cos x - \cos x \geq 0$ | $[0, 2p]$ | $\left[\mathbf{R.} \left[\frac{p}{6}, \frac{p}{2} \right] \cup \left[\frac{5p}{6}, \frac{3p}{2} \right] \right]$ |
| • $\frac{1 - \cos x}{\sqrt{3}\operatorname{tg} x - 1} < 0$ | $[-p, p]$ | $\left[\mathbf{R.} \left[-p, -\frac{5p}{6} \right) \cup \left(-\frac{p}{2}, 0 \right) \cup \left(0, \frac{p}{6} \right) \cup \left(\frac{p}{2}, p \right] \right]$ |